Matrix Representation of fuzzy Graphs

[1]Mythili.V, [2]Vasanthi.J, [3]R.Roop Rekha
[1]Associate Professor, Department of Mathematics
[2]Assistant Professor, Department of Mathematics
[1]Prince Shri Venkateshwara Padmavathy Engineering College , Chennai-600127
[2]Prince Dr.K.Vasudevan College of Engineering & Technology, Chennai.
[3]Faculty, Prince Shri Venkateshwara Arts and Science College, Gowriwakkam.

Abstact:

Although a pictorial representation of a fuzzy graph is very convenient for a visual study, other representations are better for computer processing. A matrix is a useful way of representing a graph to a computer. Here we shall discuss two most frequently used matrix representations of a fuzzy graph.

Key words: Isolated vertex, Parallel edges, Components, Permutation

1.Incidence matrix

Let G be a fuzzy graph with n vertices, e edges and no self loops. Define an nXe matrix

$A = (a_{ij})$ whose n rows correspond to n vertices and e columns correspond to e edges, as follows:

a_{ij} = min $\{v_i, e_j\}$, if edge is incident with vertex v_i

0, Otherwise.

This is called vector edge incidence matrix or simply incidence matrix and denoted by A(G).

The incidence matrix contains elements only in the range [0,1].

Observations

1.As every edge is incident on exactly two vertices, each column of A(G) has exactly two non zero entries.

2.The number of non zero entries in each row equals the degree of the corresponding vertex.

3. A row with all zeros represent the isolated vertex.

4.Parallei edges in a fuzzy graph produce identical column in it's incidence matrix.

5. If a fuzzy graph G is disconnected and consists of two components g_1 and g_2 the incidence matrix A(G) of graph G can be written in the block diagram as follows:

$$A(G) = \left[\begin{array}{c|c} A(g_1) & 0 \\ \hline 0 & A(g_2) \end{array}\right]$$

Where $A(g_1)$ and $A(g_2)$ are the incidence matrices of components g_1 and g_2 . This observation results from the fact that no edge in g_1 is incident on vertices of g_2 and vice versa. Obviously the result is true for any number of components.

Theorem 1:

Two fuzzy graphs G_1 and G_2 are isomorphic if f their corresponding incidence matrices differ only by permutations of rows and columns.

Theorem 2:

If A(G) is an incidence matrix of a connected fuzzy graph G with n vertices then rank of A(G) is n-1.

Theorem 3:

If G is a disconnected graph with n vertices and k components, then rank of A(G) is n-k.

Reduced incidence matrix:

If we remove any one row from the incidence matrix of a connected fuzzy graph, the remaining n-1 by e submatrix is of rank n-1. Such an n-1 by e submatrix A_f of A is called a reduced incidence matrix. The vertex corresponding to the deleted row of A_f is called the reference vertex. Clearly any vertex of a connected graph can be made as the reference vertex.

Sub matrices of A(G)

Let g be a subgraph of a graph G and let A(g) and A(G) be the incidence matrices of g and G respectively. Clearly A(g) is a submatrix of A(G) (possibly with rows and columns permuted). In fact there is a one-one correspondence between each n by k submatrix of A(G) and a subgraph of G with k edges , k being any positive integer less than e and n being the number of vertices in G.

Theorem 4:

Let A(G) be an incidence matrix of a connected graph G with n vertices. An n-1 by n-1 submatrix of A(G) is non singular if f the n-1 edges corresponding to the n-1 column of the matrix constitute a spanning tree in G.

Conclusion:

In this article we have discussed about various matrix representations of graphs. Incidence matrix and reduced incidence matrix was also discussed.

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